

Phasing Out PFAS: A Model of Technology Replacement and Derogation Design

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Introduction

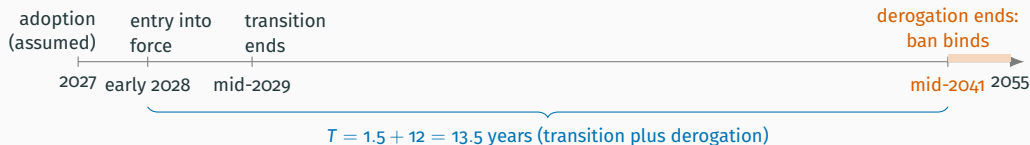
The EU PFAS ban would bind at component failure, not on a fixed date

- A near-total ban on the roughly **ten thousand PFAS**, proposed by five European governments in January 2023, moves toward implementation in 2026 (Lim 2023; European Chemicals Agency 2023; Cousins et al. 2022; European Chemicals Agency 2026)
- It bans **placing PFAS parts on the market** after a transition and derogations, while installed components **run to the end of their lives**: stringency arrives at **failure**, not at entry into force
- PFAS do not break down: releases build a **stock that never decays**, already measurable in drinking water and capitalized into housing prices (Marcus and Mueller 2024); each year of installation locks in harm

The policy question is not only whether to ban PFAS, but how fast and for how long exemptions run

Tens of billions of euros a year are at stake

- In energy, PFAS are a **building material** (seals, hoses, vessel linings, wire insulation, capacitor films): **40 to 70 percent** of in-scope equipment contains PFAS, and a PFAS-free candidate is identified for only **10 to 30 percent** of it, most not yet commercially available
- Upstream oil and gas in the EEA: about EUR 200 bn turnover and EUR 136 bn gross value added (2024); scenarios range from **routine retrofits** to **cessation of upstream activity from 2041** (Ricardo 2026)
- The ECHA risk committee (RAC) favors a near-immediate ban, the socio-economic committee (SEAC) time-limited derogations (European Chemicals Agency 2026): the corner and the interior of the choice analyzed here



Two questions, one optimal stopping model

Questions

(1) When does a ban that binds at end of life **remove the targeted substance** from service?

(2) How long should its **derogations** last?

- A facility's component fails at a **stochastic date**. At failure it replaces it with the banned part (while a derogation allows), retrofits to a substitute that arrives at a **stochastic date**, or exits irreversibly
- Ingredients: facilities heterogeneous in operating rents; a known failure hazard δ ; a substitute arriving at hazard η
- Facility policies aggregate into a **sectoral survival function**, used to choose the **derogation length T** , the regulator's instrument
- Calibrated to the EU restriction for four equipment classes of the energy sector

Preview of results

1. **Survival is a race** between failure and innovation: a retrofit-viable facility outlives the ban with probability $1 - e^{-\eta T} \delta / (\delta + \eta)$. The regulator controls less than the statute suggests
2. **The ban raises maintenance**: assessments that hold service lives fixed overstate the phase-out and its output losses
3. **Derogations are options on innovation**. With constant marginal damages an interior optimum needs $\eta > 0$ and lifecycle damages in an intermediate range; otherwise the optimum is a corner (immediate ban, or no finite ban)

At the calibrated hazards and a mid-range damage level

The proposed 13.5-year window is **longer than optimal for three of four classes** if substitutes arrive slowly, and **worse than an immediate ban for every class** if they arrive fast

Contribution: an end-of-life trigger, with the derogation as the instrument

- **Vintage-differentiated regulation:** exempting installed capital extends its life (Stavins 2006); grandfathering delays replacement (Heutel 2011; Bushnell and Wolfram 2012)
- **Investment under uncertainty:** stopping tools (Dixit and Pindyck 1994), environmental policy timing (Pindyck 2002), adoption under innovation uncertainty (Doraszelski 2004)
- **Induced innovation:** policy redirects invention (Newell et al. 1999; Popp 2002; Popp 2005)

How this paper differs

- Stringency is the **survival function of the installed stock**, with a closed form for the surviving share
- The **derogation is the instrument:** gradualism is an option on innovation, now-or-never a limit
- Closest model: adoption when the arrival rate of a new technology changes (Hagspiel et al. 2015); the contribution is institutional rather than methodological

Model

Environment: facilities, a failing component, a ban, a substitute

- Continuous time, risk neutral, discount rate $r > 0$. A unit mass of **facilities** with rent $z \sim G$ on $[\underline{z}, \bar{z}]$, fixed over time; flow profit z while running
- One critical **component**, Poisson failure hazard δ (expected life $1/\delta$); a PFAS part costs c
- The **ban**, announced at $t = 0$, bars PFAS parts from date T ; legacy parts run to failure
- A PFAS-free **substitute** arrives at hazard $\eta \geq 0$, independently across facilities; it costs $c_s = c + \kappa$, premium $\kappa > 0$
- **Exit** is irreversible and yields $s \geq 0$

A1: no exit absent the ban

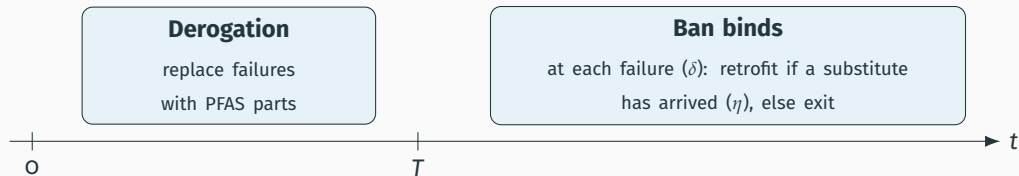
$$\underline{z} > z_0 \equiv rs + (r + \delta)c$$

A2: a positive mass in the race

$G(z_1) < 1$, with z_1 the retrofit threshold (next slides)

Each PFAS part releases ε at end of life into a stock X that never decays; damage is $D(X) = dX$ (linear in the baseline) and falls on society, not on the facility

Timing: replace before T ; after T retrofit if a substitute exists, else exit



State (z, ι, x) : rent z ; substitute indicator $\iota \in \{A, N\}$, switching $N \rightarrow A$ at hazard η ; time to the ban $x = (T - t)^+$

Choices: at each failure an action (replace with a PFAS part, or retrofit); exit time; maintenance $m_t \geq 0$ (later)

Constraints: after T the PFAS action is infeasible: a failure permits retrofit or exit if $\iota = A$, only exit if $\iota = N$

Regulator: one instrument, the date T

After the ban, facilities sort on one threshold: z_1

In words. Before T every facility replaces failures with the cheaper PFAS part. After T , with a substitute available, a facility retrofits if its rent over the exit margin, annuitized over the part's life, covers the substitute's price; otherwise it runs the legacy part down and exits at failure.

$$z_0 = rs + (r + \delta) c, \quad z_1 = rs + (r + \delta) c_s = z_0 + (r + \delta) \kappa$$
$$V_A(z) = \begin{cases} \frac{z - \delta c_s}{r}, & z \geq z_1 \\ \frac{z + \delta s}{r + \delta}, & z < z_1 \end{cases} \quad V_N(z) = \frac{z + \delta s + \eta V_A(z)}{r + \delta + \eta}$$

V_A, V_N : value of an operating facility at $t \geq T$ with / without a substitute; s exit value; c, c_s part costs.

The cost of the ban splits into a **permanent** part, the lesser of the salvage surplus a cheap part used to give and the premium κ , and a **transitory (race)** part, the retrofit surplus lost if failure beats arrival.

Survival is a race between failure and innovation

In words. A retrofit-viable facility survives if the substitute arrives during the derogation or, failing that, before its legacy part dies.

Proposition (the race)

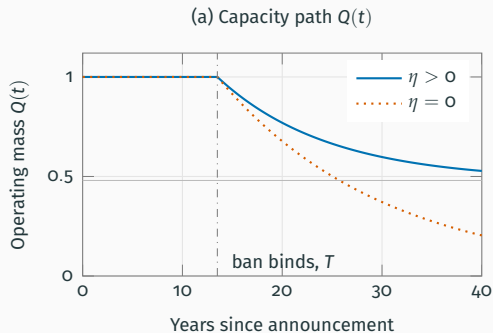
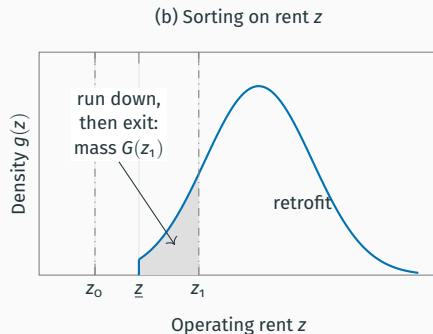
For $z \geq z_1$ (δ failure hazard, η arrival hazard, G rent distribution) the long-run operating probability is

$$\sigma(T) = 1 - \underbrace{e^{-\eta T}}_{\text{no substitute by } T} \times \underbrace{\frac{\delta}{\delta + \eta}}_{\text{failure beats arrival}}, \quad Q_\infty = [1 - G(z_1)] \sigma(T)$$

Facilities with $z < z_1$ exit at their first failure after T with probability one

- With probability $1 - e^{-\eta T}$ the substitute arrives within the derogation (T); otherwise innovation wins with probability $\eta / (\delta + \eta)$
- **Cessation** is the limit $\eta \rightarrow 0$ ($\sigma \rightarrow 0$). The regulator sets a date, but the survival curve of installed components sets the phase-in; even with immediate substitutes the ban removes mass $G(z_1) - G(z_0)$

Capacity falls as legacy parts fail; low-rent facilities run down, then exit



Notes: Stylized (levels illustrative, not calibrated). (a) Every facility operates until T ; capacity then declines as legacy parts fail, to $Q_\infty = [1 - G(z_1)]\sigma(T)$ if substitutes may arrive and to zero if not. (b) The shaded mass $G(z_1)$ runs its legacy part down and exits.

The ban lengthens the life of what it bans: over-maintenance

In words. Before the ban a failure costs a part; after it, the whole operating business. Firms maintain more (m maintenance flow, $\delta(m)$ induced hazard with $\delta' < 0$, ℓ loss per failure), the realized hazard falls, and the banned substance outlives its engineering survival curve.

$$h(\ell) = \min_{m \geq 0} \{m + \delta(m) \ell\}, \quad -\delta'(m_0) c = 1, \quad -\delta'(m_N) [V_N(z) - s] = 1$$

Proposition (over-maintenance)

If $V_N(z) - s > c$ then $m_N(z) > m_0$: the failure hazard falls and service life lengthens. This holds for every facility under A1 read at the maintained threshold. After arrival $m_A(z) > m_0$; maintenance jumps at T

- Fixed-life assessments **overstate** how fast PFAS leaves service and, among retrofit-viable facilities, output losses, as $\sigma(T)$ rises when δ falls: a 10 percent fall in the hazard adds **0.6 to 1.4 points** to $\sigma(T)$ (medium impact), about 0.3 (low)
- No source measures $\delta(m)$ for these classes: the paper reports the direction and this sensitivity

The regulator trades quasi-rents against lifecycle damage, part by part

In words. The derogation controls one margin: how long *new* PFAS parts keep entering service. Legacy parts release at end of life under any policy, so their damage is sunk.

$$\varphi = \varepsilon \frac{\delta}{r + \delta} \frac{d}{r}, \quad \Phi(T) = \varphi \delta \frac{1 - e^{-rT}}{r}, \quad W(T) = \int U_N(z, 0; T) dG(z) - \Phi(T)$$

ε release per part; $d > 0$ marginal damage per unit of stock; φ lifecycle damage of one new part; $\Phi(T)$ present value of damages from installations in $[0, T]$; W producer surplus net of damage; $U_N(z, 0; T)$ date-zero value of a facility without a substitute.

First-order condition for T (per installed part)

$$\underbrace{B}_{\text{permanent cost}} + \underbrace{e^{-\eta T} R \mathbb{E}[(z - z_1)^+]}_{\text{race cost}} = \underbrace{\varphi}_{\text{lifecycle damage}}, \quad B \equiv \mathbb{E}\left[\min\left\{\frac{z - z_0}{r + \delta}, \kappa\right\}\right], \quad R \equiv \frac{r + \eta}{r(r + \delta + \eta)}$$

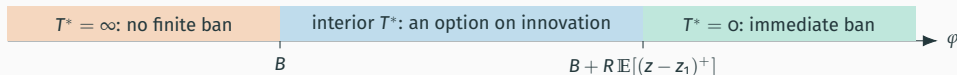
The left side falls in T : the solution is unique when interior; the race term carries $e^{-\eta T}$, the chance the derogation is still working at expiry.

Derogations are options on innovation: interior optimum, or a corner

In words. A derogation pays only if waiting buys something: probability mass on substitute arrival before forced exit.

Proposition (optimal derogation)

For $\eta > 0$ and $B < \varphi < B + R \mathbb{E}[(z - z_1)^+]$, $T^* = \frac{1}{\eta} \ln \left(\frac{R \mathbb{E}[(z - z_1)^+]}{\varphi - B} \right)$, unique and decreasing in d and ε . Corners: immediate ban ($T^* = 0$) if $\varphi \geq B + R \mathbb{E}[(z - z_1)^+]$; no finite ban ($T^* = \infty$) if $\varphi \leq B$



- If $\eta = 0$ the policy is **bang-bang**: immediate ban if $\varphi > \mathbb{E}[z - z_0] / (r + \delta)$, no finite ban if below
- The planner extends the window until the chance that no substitute has arrived falls to a damage index λ : $e^{-\eta T^*} = \lambda$

One window cannot fit classes whose substitution prospects differ

Corollary (against uniform derogations)

With classes i that differ in (η_i, κ_i) and whose optima are interior, T_i^* varies across classes: a uniform T is suboptimal unless the first-order condition holds at the same T for all. Classes sharing a corner share their optimum, and a uniform window fits them

- RO2 assigns **one derogation** to sealants, wire coatings, pipes and capacitors alike
- Classes near $\eta = 0$ face a now-or-never test; classes with active substitution prospects justify finite windows tuned to η_i

Class	Candidates	Today
Sealants	ACM, EPDM, HNBR, nickel alloy 625	30%
Wire coatings	PEEK, polyimide, ceramics	20%
Piping, hoses	C-PVC, fiber-reinforced plastic, polypropylene	10%
Capacitors	None identified	10%

Notes: Midpoints, Ricardo (2026). For capacitors no viable alternative is identified: the 10% reflects non-critical items. Most candidates are not yet commercial, so shares may be upper bounds.

Calibration to the EU restriction

Every parameter comes from the assessment

Parameter	Symbol	Value	Target
Derogation length	T	13.5 yr	18-month transition plus 12-year derogation
Discount rate	r	0.03	Real social discount rate of the assessment
Failure hazard	δ_i	0.33, 0.11, 0.16, 0.07	End-of-life flow over installed stock (upstream)
Initial substitutability	ω_i	0.30, 0.20, 0.10, 0.10	Share of equipment replaceable today
Run-down mass	$G(z_1)$	0.15 (0.10–0.20)	Closures from upgrade costs, low impact
Arrival: high impact	η	0	No substitute; activity ceases from 2041
medium	η_i	0.04, 0.04, 0.05, 0.04	45 percent activity loss: capacity 0.55
low	η_i	0.14, 0.15, 0.16, 0.16	At least 90 percent replaceable, net of ω_i
Damage per part	φ	not identified	No source reports the release per part

Notes: Classes: sealants, wire coatings, capacitors, piping. Sources: Ricardo (2026); discount rate also European Commission (2024).

Hazards: replacement rates from **engineering accounts**, not observed failures; φ is unidentified.

The arrival hazard is read from the scenarios via one equation

In words. A class has a mass $G(z_1)$ that runs down and exits; of the rest, a share ω_i already has a substitute and the remainder survives with probability $\sigma_i(T)$.

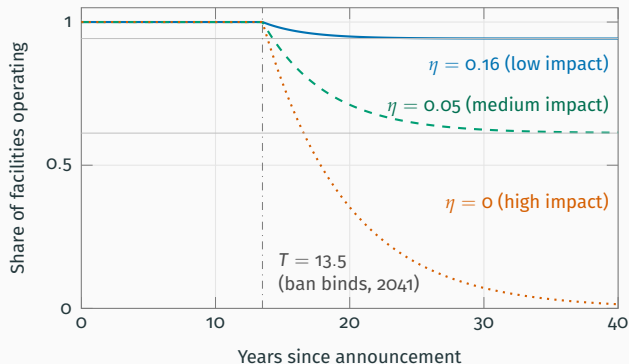
$$Q_{\infty,i} = [1 - G(z_1)] [\omega_i + (1 - \omega_i) \sigma_i(T)], \quad \sigma_i(T) = 1 - e^{-\eta_i T} \frac{\delta_i}{\delta_i + \eta_i}$$

- **High:** $\eta = 0$. Cessation is not a class-level outcome: it follows from a critical class with no substitute, capacitors, through the product rule
- **Medium** (45 percent loss): $Q_{\infty,i} = 0.55$ gives $\eta_i = 0.04, 0.04, 0.05, 0.04$
- **Low:** $1 - (1 - \omega_i)e^{-\eta_i T} = 0.90$ gives $\eta_i = 0.14, 0.15, 0.16, 0.16$; the 15 percent activity loss is then a model output

Keep in view

Every class gets the same target, so η_i differ only through δ_i, ω_i ; targets are long-run levels; each class is matched alone (all four together: 0.11 at the medium values)

The arrival hazard alone separates a managed transition from collapse



- Capacitors, the class with no viable alternative: $\delta = 0.16$
- Long-run survival: 0.94, 0.61, 0 for low, medium, high impact
- Curves are **transitional paths**, not steps: in 2055 the medium share is 0.63; at high impact 0.11 (capacitors), 0.38 (piping), 0.01 (sealants)

Notes: Operating share of facilities above z_1 without an initial substitute (an initial share ω lifts each curve to $\omega + (1 - \omega)\sigma$); capacitor class, $\delta = 0.16$, $T = 13.5$. The regulator's date is common to all paths.

Calibrated long-run survival is 0.48 to 0.63 (medium), 0.89 to 0.96 (low)

	Sealants	Wire coatings	Capacitors	Piping
Failure hazard δ_i (per year)	0.33	0.11	0.16	0.07
Replaceable today, ω_i	0.30	0.20	0.10	0.10
<i>Medium impact</i> (assessment: activity –45%)				
Arrival hazard η_i	0.04	0.04	0.05	0.04
Substitute available by T , $1 - (1 - \omega_i)e^{-\eta_i T}$	0.59	0.53	0.54	0.48
Conditional survival $\sigma_i(T)$	0.48	0.57	0.61	0.63
Long-run class capacity $Q_{\infty,i}$	0.54	0.56	0.55	0.57
<i>Low impact</i> (assessment: activity –15%)				
Arrival hazard η_i	0.14	0.15	0.16	0.16
Substitute available by T	0.89	0.89	0.90	0.90
Conditional survival $\sigma_i(T)$	0.89	0.94	0.94	0.96
Long-run class capacity $Q_{\infty,i}$	0.79	0.81	0.81	0.82

Notes: $T = 13.5$, $r = 0.03$, $G(z_1) = 0.15$. Medium η_i matches capacity 0.55; low η_i solves $1 - (1 - \omega_i)e^{-\eta_i T} = 0.90$ (low capacity is a model output). $\sigma_i(T)$: retrofit-viable facility, no initial substitute.

The 13.5-year window is too long for three classes (slow substitutes)

	Sealants	Wire coatings	Capacitors	Piping
<i>Medium impact</i>				
Optimal derogation T^* at $\lambda = \frac{1}{2}$ (years)	8.4	11.8	11.8	14.7
λ matching RO2's $T = 13.5$	0.41	0.47	0.46	0.52
<i>Low impact</i>				
Optimal derogation T^* at $\lambda = \frac{1}{2}$ (years)	2.4	3.1	3.7	3.7
λ matching RO2's $T = 13.5$	0.11	0.11	0.10	0.10

- $\lambda \equiv (\varphi - B) / (R \mathbb{E}[(z - z_1)^+])$ is the **net lifecycle damage as a share of the retrofit surplus at risk**; with initial substitutes $(1 - \omega)e^{-\eta T^*} = \lambda$, so $T^* = \eta^{-1} \ln[(1 - \omega) / \lambda]$
- $\lambda = \frac{1}{2}$ is a **reference point, not an estimate**; a more damaging class draws a shorter window. RO2's window is optimal for a class at one index value only: about 0.10 (low), 0.41 to 0.52 (medium)

The cost of the proposal lies in its length, not in its uniformity

Loss as a share of the value the class's optimal derogation delivers over an immediate ban ($\lambda = \frac{1}{2}$); the best common window minimizes the sum of these shares.

- **Medium impact:** best common window 10.8 years, at most 6 percent lost to uniformity. The 13.5-year window costs about 1 percent for wire coatings and capacitors, 23 percent for sealants (mostly length); only piping would take a slightly longer one
- **Low impact:** best common window 3.0 years, at most 5 percent. The 13.5-year window forgoes 3 to 9 times the derogation's value: **worse than an immediate ban** for every class
- **Mixed prospects** (low hazard for sealants and wire coatings, medium for capacitors and piping): best common window 3.2 years, forgoing about half the value for capacitors and piping; 13.5 years is worse than an immediate ban for sealants and wire coatings
- Within either scenario every class has nearly the same hazard, so uniformity costs little; it **becomes costly when prospects differ**. All rankings are conditional on the damage index

Now or never: when does an immediate ban beat any derogation?

In words. With $\eta = 0$ an immediate ban dominates iff the annual lifecycle damage of continued installation exceeds the fraction $\delta / (r + \delta)$ of average quasi-rents.

- At $r = 0.03$ the fraction is 0.84 for capacitors, 0.70 to 0.92 across classes
- With upstream value added at risk (EUR 18 bn per year, high impact, PV) as quasi-rents, the test passes if lifecycle damage of a year of installation exceeds **EUR 15 bn** at the capacitor hazard (12.6 to 16.5 bn across classes)
- On a flow basis, $\delta d\varepsilon > r \mathbb{E}[z - z_0]$: a year of installations must add more than $0.03 \times 18 =$ **EUR 0.54 bn** of permanent annual damage, about **1.4 percent** of the Commission's central estimate of annual health costs of four legacy PFAS (EUR 39.5 bn, 2024) and 0.6 to 1.0 percent of an earlier range (EUR 52 to 84 bn) (European Commission 2026; Goldenman et al. 2019)

A scale and nothing more

Value added exceeds quasi-rents; the 18 bn is an annualized present value; the aggregates concern legacy PFAS and no source measures release from fluoropolymer equipment or its marginal damage. Whether the test passes turns on two magnitudes the record does not report

Comparative statics and extensions

Comparative statics: more damage or more critical parts shorten the window

	λ	Seal.	Wire	Cap.	Pipe
Medium	0.25	25.7	29.1	25.6	32.0
	0.50	8.4	11.8	11.8	14.7
	0.75	0	1.6	3.6	4.6
Low	0.25	7.4	7.8	8.0	8.0
	0.50	2.4	3.1	3.7	3.7
	0.75	0	0.4	1.1	1.1

Notes: Optimal derogation T^* in years; zero is an immediate ban.

- Higher damage (d, ε, λ): the interior T^* **falls** and the immediate-ban region widens
- Initial substitutes ω : T^* falls by $\eta^{-1} \ln[1/(1-\omega)]$, between 0.7 and 8.9 years
- n critical parts per unit: the matched η rises and the window shortens by 1 year (sealants) to 6 (piping) at $n = 10$

Medium impact: T^* is 25.6 to 32.0 years at $\lambda = 0.25$ and 0 to 4.6 years at $\lambda = 0.75$

Extensions keep the structure of the results

- **Convex damages:** a second, stock-based motive; with $D'' > 0$ an interior window can be optimal even at $\eta = 0$ [Backup](#)
- **Maintenance:** both damage-side corrections shorten the window; sized with a borrowed technology at mid-range damage, by 0.2 to 0.3 years (sealants) and 1.9 to 4.0 (piping) [Backup](#)
- **Several and heterogeneous components:** exact reduction to an effective hazard $n_i \delta_i$; facility survival is the product across classes [Backup](#)
- **Global welfare:** a global criterion raises $\varphi \rightarrow \varphi_g$ and shortens T^* ; exit that relocates production lengthens it; the model does not sign the net effect
- **First-best:** an end-of-life charge decentralizes it; the derogation can match it only at the corners [Backup](#)
- **Spare-parts derogation:** a second instance of the instrument; where it applies, T is the later date and every result holds at it

Policy implications and conclusion

Policy: differentiate derogations by class, tuned to two hazards

- **Differentiate by component class**, tuned to each arrival hazard: RO2's one window for seals, wire coatings, pipes and capacitors is the design the model warns against
- In the calibration **length matters more than uniformity**. Both rankings depend on a damage level the record does not identify: at higher damage the same window is too long even for slow substitution; at lower damage it beats an immediate ban even for fast substitution
- Classes with **no substitution prospects** and stable marginal damages face a now-or-never test: damages against quasi-rents
- Impact assessments should treat **service lives as choices**: the ban itself raises maintenance and slows the phase-out
- Where a charge on releases is available it **decentralizes the first-best**; the derogation length is then the second-best instrument. The two primitives, failure and arrival, are observable in principle

Conclusion: the regulator fixes a date; failure and innovation fix the outcome

Findings

1. Survival under an end-of-life ban is a **race** between failure and innovation
2. The ban **raises maintenance**; fixed service lives overstate the phase-out
3. A derogation buys the probability that a substitute arrives before the installed base dies: an **option on innovation**

One window is too short where substitutes are far off and, at mid-range damage, can be worse than a ban where they are near

Caveats the paper states

- Rankings depend on a damage level the record does not identify
- The calibration translates an industry-commissioned assessment; η is not identified from outside it
- Single-component benchmarks from engineering accounts; exponential lifetimes rule out wear-out
- Partial equilibrium at a fixed output price; the joint facility problem across classes is left to future work

Appendix

Backup: notation

Symbol	Meaning
$z \sim G$	Facility operating rent
r	Discount rate
δ	Component failure hazard (life $1/\delta$)
η	Substitute arrival hazard
$c, c_s = c + \kappa$	PFAS and substitute part cost
s	Exit value (scrap and relocation)
$m, \delta(m)$	Maintenance flow and induced hazard
$h(\ell)$	Minimized maintenance plus expected loss
T	Derogation length; the ban binds at T
$z_0 = rs + (r + \delta)c$	Operate threshold
$z_1 = z_0 + (r + \delta)\kappa$	Retrofit threshold

Symbol	Meaning
ε	Release per PFAS part at end of life
$X, D(X)$	Release stock (no decay); damage, $d = D'$
φ	Lifecycle damage of one installed part
$\Phi(T)$	PV of lifecycle damages
θ	Domestic share of marginal damage
φ_g, τ^*	Global damage per part; first-best charge $d_g \varepsilon / r$
$\sigma(T), Q(t)$	Long-run operating share; capacity path
$\Delta_i = \sum_k \delta_{ik}$	Class effective hazard
λ, B, R	Damage index; cost and race-surplus terms
ω	Share starting with a substitute

Backup: the three optimization problems

P1, the facility. State (ι, x) , $\iota \in \{N, A\}$, $x = (T - t)^+$; controls: maintenance $m_t \geq 0$, an action at each failure (replace, feasible iff $x > 0$; retrofit, feasible iff $\iota = A$) and an exit time τ_x (irreversible, value s). Failures arrive at intensity $\delta(m_t)$; ι switches $N \rightarrow A$ at η .

$$V(z) = \sup \mathbb{E} \left[\int_0^{\tau_x} e^{-rt} (z - m_t) dt - \sum_{\tau_k < \tau_x} e^{-r\tau_k} c_{a_k} + e^{-r\tau_x} s \right]$$

P2, the regulator. Commits at $t = 0$ to one scalar $T \in [0, \infty)$, taking the competitive equilibrium as constraint; maximizes $W(T) = \int U_N(z, 0; T) dG(z) - \Phi(T)$. Corners: $T = 0$ and $T \rightarrow \infty$

P3, the planner. Chooses the full contingent allocation (action, maintenance) at state (z, j, ι) , $j \in \{P, S\}$ the material of the part in service; no ban, and each end of life of a PFAS part costs $d_g \varepsilon / r$. Block recursivity separates the problem across facilities

Backup: values and costs of the ban

- **State A.** The replacing branch solves $rV_A = z - \delta c_s$, consistent iff $V_A - c_s \geq s$, i.e. $z \geq z_1$. The run-down branch gives $V_A = (z + \delta s) / (r + \delta)$ for $z < z_1$. The branches meet at z_1 with value $s + c_s$
- **State N.** For $z < z_1$, $V_N = V_A$: a firm that will not buy the substitute gains nothing from its arrival. For $z \geq z_1$,

$$V_A - V_N = \frac{\delta [V_A - s - c_s]}{r + \delta + \eta} = \frac{\delta (z - z_1)}{r(r + \delta + \eta)} \geq 0$$

- **Cost of the ban.** $\Delta_A = -\frac{\delta}{r} \min\{\frac{z - z_0}{r + \delta}, \kappa\}$ (the two branches meet at z_1 since $z_1 - z_0 = (r + \delta)\kappa$);
 $\Delta_A - \Delta_N = V_A - V_N = \frac{\delta (z - z_1)^+}{r(r + \delta + \eta)}$
- **Before the ban** ($x = T - t$):

$$U_A = \frac{z - \delta c}{r} + e^{-rx} \Delta_A, \quad U_N = \frac{z - \delta c}{r} + \Delta_A e^{-rx} + [\Delta_N - \Delta_A] e^{-(r+\eta)x}$$

a permanent component discounted at r and a transitory one discounted at $r + \eta$, because arrival extinguishes it

Backup: derivation of the survival probability

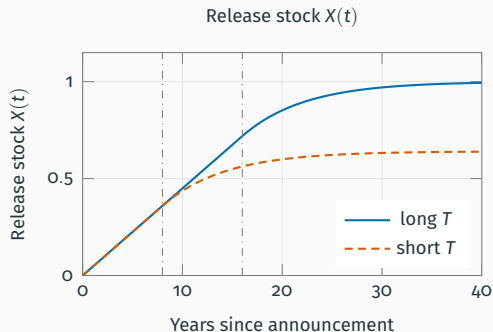
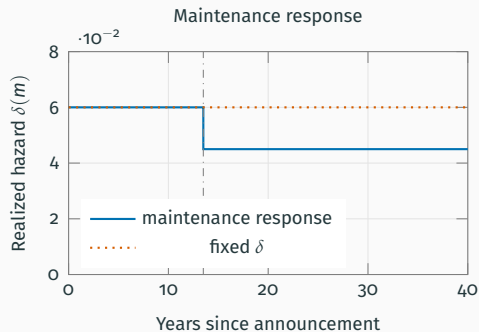
A retrofit-viable facility, $z \geq z_1$, measured from the ban date, operates in the long run in either of two disjoint events.

- With probability $1 - e^{-\eta T}$ the substitute has arrived by T : it retrofits at every failure and operates forever
- With probability $e^{-\eta T}$ none has arrived: it operates forever only if arrival beats the next failure, probability $\int_0^\infty \eta e^{-\eta u} e^{-\delta u} du = \eta / (\delta + \eta)$

$$\sigma(T) = (1 - e^{-\eta T}) + e^{-\eta T} \frac{\eta}{\delta + \eta} = 1 - e^{-\eta T} \frac{\delta}{\delta + \eta}$$

For $z < z_1$ the firm exits at its first failure after T in either state. Long-run mass under the ban is $[1 - G(z_1)]\sigma(T)$ against 1 absent the ban; the loss due to the premium alone is the limit $\sigma \rightarrow 1$, namely $G(z_1)$ (A1 implies $G(z_0) = 0$).

Backup: release stock and hazard response



- The stock rises at the constant rate $\delta\epsilon$ until T , then levels off as the parts installed at T fail: a **longer derogation raises the permanent plateau**
- The realized hazard falls from $\delta(m_O)$ to $\delta(m_N)$ at T ; schematic, the size of the fall is not calibrated

Backup: optimal derogation with endogenous maintenance

Proposition (optimal derogation with endogenous maintenance)

With A1 at z_0^m , class welfare is

$$W^m(T) = K^m + \underbrace{(\Gamma_A - P) e^{-rT}}_{\text{damage avoided net of permanent cost}} - \underbrace{(S - \Gamma_N) e^{-(r+\eta)T}}_{\text{race cost net of release delay}}$$

For $\eta > 0$, $S > \Gamma_N$ and $0 < r(\Gamma_A - P) < (r + \eta)(S - \Gamma_N)$: $T^{m*} = \eta^{-1} \ln[(r + \eta)(S - \Gamma_N) / (r(\Gamma_A - P))]$. Immediate ban if $r(\Gamma_A - P) \geq (r + \eta)(S - \Gamma_N)$; no finite ban if $\Gamma_A \leq P$

- P permanent cost; $S = \mathbb{E}[V_A - V_N]$ race cost; Γ_A damage avoided by ending installations; $\Gamma_N \geq 0$ release delay bought by maintenance while waiting for a substitute; K^m constant in T
- At a fixed hazard $P = \frac{\delta}{r} B$, $\Gamma_A = \frac{\delta}{r} \varphi$, $\Gamma_N = 0$, and T^{m*} is the baseline T^*
- Both **damage-side corrections shorten** the derogation; the private-side corrections have no general sign. With no measured technology, the calibration keeps the fixed-hazard formula: its T^* is a benchmark, not a signed bound

Backup: derivation of $W(T)$, the first-order condition and T^*

- **Lifecycle damage:** a part fails at $\vartheta \sim \text{Exp}(\delta)$ and releases ε , of PV $d\varepsilon/r$ at release:
 $\varphi = \mathbb{E}[e^{-r\vartheta}] d\varepsilon/r = \frac{\delta}{r+\delta} \frac{d\varepsilon}{r}$; installations flow at rate δ in $[0, T]$, so $\Phi(T) = \int_0^T e^{-rt} \delta \varphi dt$

- **Welfare:** integrating U_N over G and subtracting $\Phi(T)$, with $\mathbb{E}[\Delta_A] = -\frac{\delta}{r}B$ and $\mathbb{E}[\Delta_N - \Delta_A] = -\frac{\delta}{r}\mathbb{E}[(z - z_1)^+]/(r + \delta + \eta)$,

$$W(T) = K + \frac{\delta}{r}(\varphi - B)e^{-rT} - \frac{\delta}{r} \frac{\mathbb{E}[(z - z_1)^+]}{r + \delta + \eta} e^{-(r+\eta)T}$$

- **FOC:** $\partial U_N(z, 0; T)/\partial T = e^{-rT}\{r[-\Delta_A] + (r + \eta)[\Delta_A - \Delta_N]e^{-\eta T}\}$. Aggregate, set equal to $\Phi'(T) = \varphi\delta e^{-rT}$, divide by δe^{-rT} and use $r[-\Delta_A] = \delta \min\{\cdot\}$, $(r + \eta)[\Delta_A - \Delta_N] = \delta R(z - z_1)^+$: $B + e^{-\eta T}R\mathbb{E}[(z - z_1)^+] = \varphi$
- **Optimum:** the left side is continuous and decreasing in T (strictly under A2), from $B + R\mathbb{E}[(z - z_1)^+]$ at $T = 0$ to B as $T \rightarrow \infty$; solve $e^{-\eta T^*} = (\varphi - B)/(R\mathbb{E}[(z - z_1)^+])$
- **Case $\eta = 0$:** $R = 1/(r + \delta)$ and $\min\{\frac{z - z_0}{r + \delta}, \kappa\} + \frac{(z - z_1)^+}{r + \delta} = \frac{z - z_0}{r + \delta}$ (as $z_1 - z_0 = (r + \delta)\kappa$): the left side is the constant $\mathbb{E}[z - z_0]/(r + \delta)$ and the policy is bang-bang

Backup: convex damages and the second motive for gradualism

With a general damage function ($D' > 0$, $D'' \geq 0$), the option value of innovation sits on the left and dies with η ; the timing value of a small stock sits on the right and dies with D'' :

$$B + (1 - \omega) e^{-\eta T} R \mathbb{E}[(z - z_1)^+] = \varphi(T), \quad \varphi(T) \equiv \varepsilon \int_0^\infty e^{-ru} (1 - e^{-\delta u}) D'(X(T + u; T)) du$$

$X(\cdot; T)$ stock path induced by T ; $\varphi(T)$ lifecycle damage of a part installed at expiry; ω share of facilities starting with a substitute. With linear damages $\varphi(T) = \varphi$.

Proposition (two motives for gradualism)

Let $\eta = 0$, $\omega = 0$. If $D'' = 0$ the policy is bang-bang. If $D'' > 0$, $\varphi(\cdot)$ is strictly increasing while installations continue, and a unique interior T^* solving $\varphi(T^*) = \mathbb{E}[z - z_0]/(r + \delta)$ exists whenever $\varphi(0) < \mathbb{E}[z - z_0]/(r + \delta) < \lim_{T \rightarrow \infty} \varphi(T)$

- With constant marginal damages only innovation can justify a window; when the stock raises the price of a release, now-or-never **softens into a finite date**
- Innovation that has already happened is not worth waiting for: a share ω with a substitute lowers the interior optimum by $\eta^{-1} \ln[1/(1 - \omega)]$

Backup: several components and heterogeneous lives

Proposition (reduction to an effective hazard)

Within class i a facility with n_i identical components solves the single-component problem with δ_i replaced by $n_i\delta_i$; thresholds $z_0(n_i) = rs + (r + n_i\delta_i)c$, $z_1(n_i) = z_0(n_i) + (r + n_i\delta_i)\kappa$. A facility retrofit-viable in every class it carries operates in the long run with probability

$$S(T) = \prod_i \sigma_i(T; n_i), \quad \sigma_i(T; n_i) = 1 - e^{-\eta_i T} \frac{n_i \delta_i}{n_i \delta_i + \eta_i}$$

- Treating the first failure as fatal matches a critical component; where partial operation is feasible the effective hazard is lower, so the assumption bounds survival from below
- Treating each class as its own population of single-component facilities, as the calibration does, **overstates facility survival** and understates the output loss
- Heterogeneous lives collapse to a sum: the first failure has rate $\Delta_i = \sum_{j \in i} \delta_j$, and the proposition holds with Δ_i ; only the per-part damage $\varphi_j = \varepsilon \frac{\delta_j}{r + \delta_j} \frac{d}{r}$ depends on a part's own hazard
- The thresholds describe a facility whose critical parts lie in one class; the joint-facility problem across classes is left to future work

Backup: first-best charge versus second-best derogation

	First-best: end-of-life charge $\tau^* = d_g \varepsilon / r$	Second-best: derogation length T
Instrument	Charge on each PFAS part at end of life	Date after which PFAS parts are barred
Conditions on	Facility rent z and substitute availability	The calendar date alone
Decision rule	Install iff surplus covers φ_g and, once a substitute exists, $\varphi_g \leq \kappa$	Install iff $t < T$, for every z
Extensive margin	Efficient sorting	Admits parts the charge prices out before T ; bars parts it admits after T
Maintenance Welfare (global)	Priced by the charge W^{FB}	Unpriced; distortion of either sign $W_g(T) \leq W^{\text{FB}}$, equal only at the corners

- The charge implements the first-best, maintenance included; its value at installation is $\varphi_g = \frac{\delta}{r+\delta} \tau^*$, which reproduces exit and retrofit decisions at a fixed hazard but not maintenance
- A REACH restriction sets no per-unit charge, so the date is the margin it controls

Backup: failure hazard by class and sector

Class	Sector	Installed stock	End of life per year	Hazard δ_{ij}
Sealants (units)	Upstream	26,000–52,000	8,500–17,500	0.333 (0.327, 0.337)
	CCS	3,200–4,600	1,050–1,550	0.333 (0.328, 0.337)
	Refining	18,000–66,000	3,000–15,000	0.214 (0.167, 0.227)
	Distribution	1,740–3,450	530–1,200	0.333 (0.305, 0.348)
Wire coatings (km)	Upstream	40–80	4–9	0.108 (0.100, 0.113)
	CCS	5–10	0.5–1.5	0.133 (0.100, 0.150)
	Refining	140–570	10–40	0.070 (0.071, 0.070)
	Distribution	153–256	15–30	0.110 (0.098, 0.117)
Capacitors (units)	Upstream	1,000–3,000	150–500	0.163 (0.150, 0.167)
	CCS	570–2,200	90–370	0.166 (0.158, 0.168)
	Refining	4,300–16,500	450–1,800	0.108 (0.105, 0.109)
	Distribution	3,650–6,840	575–895	0.140 (0.158, 0.131)
Piping and hoses (km)	Upstream	400–550	25–40	0.068 (0.062, 0.073)
	CCS	460–620	3–5	0.007 (0.007, 0.008)
	Refining	300–500	5–25	0.037 (0.017, 0.050)
	Distribution	186–346	11–27	0.071 (0.059, 0.078)

Notes: Typical operation (Ricardo 2026); bracket: ratio of lower and of upper bounds. Calibration: upstream values, rounded. CCS piping is an unexplained outlier, not used.

Backup: external check on the arrival hazard

Reading a scenario's replaceable share p as the cumulative arrival $1 - e^{-\eta T}$ gives $\eta = -\ln(1 - p)/T$, a transparent bound rather than the full inversion behind the baseline.

Scenario	Share p , midpoint (range)	Implied η , midpoint (range)	Calibrated η
High impact	0.20 (0.10, 0.30)	0.017 (0.008, 0.026)	0
Medium impact	0.40 (0.20, 0.60)	0.038 (0.017, 0.068)	0.04–0.05
Low impact	≥ 0.90	≥ 0.171	0.14–0.16

- The medium values lie inside the implied range, which spans a factor of four: the check bounds η rather than pinning it; it shows internal consistency, not an independent estimate
- The implied mean time to a substitute, $1/\eta$, is 20 to 25 years at the medium values and about 6 to 7 years at the low values
- The high value is the cessation corner: the evidence supports $\eta = 0$ for capacitors only; zero for the other three classes is an added pessimistic assumption of the scenario

Backup: other readings of the target; several critical parts per unit

Reading of the target (η_i / T^* at $\lambda = \frac{1}{2}$)	Sealants	Wire coatings	Capacitors	Piping
Long-run class capacity (baseline)	0.042 / 8.0	0.038 / 12.2	0.049 / 11.9	0.038 / 15.7
Class capacity in 2055	0.041 / 8.2	0.026 / 18.4	0.043 / 13.6	0.015 / 38.6
Average class capacity, 2041–2055	0.022 / 15.5	none	0.015 / 39.9	none
Conditional survival $\sigma_i(T)$	0.049 / 14.2	0.037 / 18.5	0.042 / 16.5	0.032 / 22.0

Critical parts per unit (η_i / T^*)	$n = 1$	$n = 2$	$n = 5$	$n = 10$
Sealants	0.042 / 8.0	0.046 / 7.4	0.049 / 6.9	0.050 / 6.8
Wire coatings	0.038 / 12.2	0.046 / 10.1	0.054 / 8.8	0.057 / 8.3
Capacitors	0.049 / 11.9	0.057 / 10.3	0.064 / 9.2	0.066 / 8.9
Piping	0.038 / 15.7	0.048 / 12.3	0.058 / 10.1	0.063 / 9.3

- Finite-horizon readings imply lower hazards and longer windows, as parts still in service keep capacity above its long-run level; for wire coatings and piping the average reading has no solution (average capacity is 0.59 and 0.67 even with no substitute)
- Unrounded hazards, so the baseline row differs slightly from the main table (rounded to two decimals); matched to the same long-run target at $G(z_1) = 0.15$

Backup: welfare cost of the 13.5-year window across damage indices

Scenario	λ	Sealants	Wire coatings	Capacitors	Piping
Medium	0.25	0.14	0.17	0.13	0.20
	0.40	0.00	0.03	0.02	0.07
	0.50	0.23	0.01	0.01	0.00
	0.60	4.03	0.49	0.27	0.07
	0.75	ban	35.10	4.50	2.47
Low	0.25	0.28	0.21	0.17	0.17
	0.40	2.41	1.49	1.06	1.06
	0.50	9.33	4.56	2.84	2.84
	0.60	56.74	15.55	7.61	7.61
	0.75	ban	404.23	49.24	49.24

- Cost of the window $T_u = 13.5$ as a share of the value of each class's optimal derogation over an immediate ban; above one, the window is worse than an immediate ban
- “ban”: the class's own optimum is an immediate ban. Shares are large when the class optimum is short, because the value of its optimal derogation is then small

Backup: loss decomposed into uniformity and length ($\lambda = \frac{1}{2}$)

Uniformity / length	Sealants	Wire coatings	Capacitors	Piping
Medium (common window 10.8)	0.06 / 0.18	0.00 / 0.01	0.00 / 0.01	0.05 / -0.04
Low (common window 3.0)	0.05 / 9.28	0.00 / 4.56	0.03 / 2.81	0.03 / 2.81
Mixed (common window 3.2)	0.09 / 9.25	0.00 / 4.56	0.47 / -0.45	0.55 / -0.54

- Uniformity is the loss at the best common window, which minimizes the sum of the four shares; length is the further loss from setting the common window at 13.5 years
- A negative length means the statutory window is closer to that class's optimum than the best common window is
- Mixed: low-impact hazard for sealants and wire coatings, medium-impact hazard for capacitors and piping

Backup: sizing the maintenance corrections with a borrowed technology

	Estimates (Albonico et al. 2014)					Elast.	Hazard relative to pre-ban		
	γ	θ	ψ	ξ	M/K		$\rho = 2$	$\rho = 5$	$\rho \rightarrow \infty$
Canada	19.19	2.25	0.52	0.19	0.0494	-0.56	0.77	0.61	0.50
United States	8.79	2.73	0.58	0.16	0.0309	-0.26	0.69	0.47	0.31

Class	Fixed hazard			Canada		United States	
	δ_i	η_i	T^*	(a)	(b)	(a)	(b)
Sealants	0.33	0.04	8.4	8.2	8.2	8.1	8.1
Wire coatings	0.11	0.04	11.8	10.8	10.6	10.1	9.6
Capacitors	0.16	0.05	11.8	11.3	11.3	10.9	10.8
Piping	0.07	0.04	14.7	12.8	12.3	11.7	10.7

- Transfer assumption: a class's hazard responds to a rise in the loss from a failure in the same proportion as aggregate equipment depreciation. (a): hazard at the floor only while waiting for the substitute; (b): in both post-ban states
- Private coefficients at fixed-hazard values and $B = 0$, $\lambda = \frac{1}{2}$: the numbers **bound the damage-side corrections**; they are not a corrected optimum and not part of the calibration

Backup: machine-checked results

- The results of the proofs appendix are also machine-checked in Lean 4 (Moura and Ullrich 2021) with Mathlib (The mathlib Community 2020): **85 theorems, no sorry**; each depends only on the axioms `propext`, `Classical.choice` and `Quot.sound`
- Checks are definitional identities, algebraic checks of closed forms, or verified analytic results (e.g. the closed-form T^* , the global optimum of W , uniqueness of the state- A value)

Three limits of scope

(1) Expectations over the rent distribution enter as parameters, so the welfare forms are definitions in the development; their derivation by integration is on paper. (2) Two checks are conditional: the interior root under convex damages (given a positive derivative of lifecycle damage), and the distinct optima of the uniform-window corollary. (3) The first-best identity holds by the definition of τ^* : it checks the form of the Bellman system, not the existence of its solution

- Established **on paper only**: the population aggregation of the equilibrium result, the measure-zero form of the uniform-window corollary, the fixed point of V_N with endogenous maintenance, and the solution of the first-best system with endogenous maintenance

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